

7.5 Roots and Zeros

Fundamental Theorem of Algebra

- all polynomial equations with a degree greater than zero have at least one root in the set of complex numbers
- a polynomial equation of the form $P(x) = 0$ of degree n with complex coefficients, has exactly n roots

How many roots?

ex:

$$a - 10 = 0$$

1

$$x^2 + 2x - 48 = 0$$

2

$$3a^3 + 18a = 0$$

3

$$y^5 - y = 0$$

5

Solve # 3.

$$3a^2 + 18a = 0$$

$$3a(a^2 + 6) = 0$$

$$3a = 0 \quad a^2 + 6 = 0$$

$$a = 0 \quad a^2 = -6$$

$$a = \pm i\sqrt{6}$$

1 R
2 Imaginary

Solve # 4.

$$y(y^4 - 1) = 0$$

$$y(y^2 + 1)(y^2 - 1) = 0$$

$$y = 0 \quad y^2 = -1 \quad y^2 = 1$$

$$y = 0 \quad y = \pm i \quad y = \pm 1$$

3 R
2 Imag

Complex Conjugates

$$a, b \in \mathbb{R} \quad b \neq 0$$

If $a + bi$ is a zero, then $a - bi$ is a zero

1. If $3 + 2i$ is a zero, then $3 - 2i$ is also a zero.

2. If $5 - 3i$ is a zero, then $5 + 3i$ is also a zero.

3. If $\sqrt{5}$ is a zero, then $1 + \sqrt{5}$ is also a zero.
(It also works for radicals)

4. If $6i$ is a zero, then $-6i$ is also a zero.

Write a function with the following zeros.

3, $2 + i$, $2 - i$

$$\begin{aligned} \text{sum} &= 4 \\ \text{prod} &= 5 \\ x^2 - \text{sum } x + \text{product} &= 0 \end{aligned}$$

$$\begin{aligned} (x-3)(x^2 - 4x + 5) &= 0 \\ x^3 - 7x^2 + 17x - 15 &= 0 \end{aligned}$$

Write a function with the following zeros.

-2, $2 - \sqrt{3}$, $2 + \sqrt{3}$

$$\begin{aligned} \text{sum} &= 4 \\ \text{prod} &= 1 \\ (x+2)(x^2 - 4x + 1) &= 0 \\ x^3 - 2x^2 - 7x + 2 &= 0 \end{aligned}$$

Write a function with the following zeros.

-8, $5i$

HW

p375-376

13, 15, 17,

35, 38, 39, 41